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ORIGINAL PAPER

A solution algorithm for finding the best and the worst fuzzy compromise solutions of fuzzy rough linear programming problem with triangular fuzzy rough number coefficients

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Abstract

An algorithm for the solution of the fuzzy rough linear programming (FRLP) problems with triangular fuzzy rough number parameters is proposed to overcome the uncertainty in decision-making problems. While the costs of the objective function and the coefficients of constraints are triangular fuzzy rough numbers, the variables are in the triangular fuzzy form. Fuzzy optimal solutions are found by solving two distinct fully fuzzy linear programming (FFLP) problems, obtained from the lower and upper approximations of the FRLP problem. These solutions are utilized to evaluate different fuzzy objective function values, and these values are compared according to their supports to specify the worst and the best fuzzy rough compromise solutions. The decision-maker (DM) can make a final decision within the bounds of these supports according to the direction of the optimization. Also, the algorithm yields approximate fuzzy rough compromise solutions in the infeasibility case of the FFLP problems. To demonstrate the efficiency of the algorithm, some numerical examples are illustrated.

Keywords Fuzzy linear programming problems · Fuzzy rough numbers · Triangular fuzzy numbers · Compromise solution

1 Introduction

Linear programming (LP) is a type of conventional mathematical programming problem used in modeling real-life problems and has a wide application area in mathematics. It is generally used in decision-making and optimization problems. An LP problem deals with finding one or more (i.e., alternative) optimal solutions under linear constraints and aims to maximize or minimize a linear objective function. When the coefficients of constraints and the costs of objective function are crisp numbers, these LP problems are solved via classical methods. Since the data collected from the real-life may contain some errors or have uncertainty arising from lack of information, the coefficients and variables cannot be expressed precisely. Therefore, it is more appropriate to state these variables and parameters by fuzziness, roughness, or the hybrid form of fuzziness and randomness, called fuzzy rough values.

Fuzziness was introduced by Zadeh (1965) in 1965 to deal with imprecision or ambiguity in real-life human perception. In the fuzzy set theory, an element has a membership value representing the degree of belonging to a set. Triangular, trapezoidal, and LR-type fuzzy numbers are special fuzzy sets mostly used. For example, a triangular fuzzy number is a triplet value. Any fuzzy event can be represented using the left, middle, and right components which are corresponding to the smallest likely, probable, and the largest possible values, respectively. A fuzzy LP problem, which has at least one fuzzy parameter, can be solved via some methods proposed in (Cadenas and Verdegay 1997; Dong and Wan 2018; Fang et al 1999; Jiménez et al 2007; Liu 2001; Maleki et al 2000; Wan and Dong 2014; Zimmermann 1978). If all the parameters of an LP problem are fuzzy, then it is called a fully fuzzy LP problem (FFLP), and the methods such as (Akram et al 2022; Allahviranloo et al 2008; Buckley and Feuring 2000; Ezzati et al 2015; Ganesan and Veeramani 2006; Khan et al 2013; Kumar et al 2011; Kumar and Kaur 2013) can

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be used to solve. Pawlak (1982) presented an approach as an alternative to the fuzzy theory and tolerance theory in 1982 and named this study “Rough Sets”. Rough sets are applied with crisp intervals and fuzzy sets. Rough set theory expresses the vagueness by defining a boundary region of a set. Accordingly, in the rough set theory, a vague concept is represented by a pair of precise concepts known as the lower and the upper approximations. The lower approximation contains all elements which are surely in the vague concept, whereas the upper approximation is contained of all elements which possibly belong to the concept. Rough concept has a great importance in the areas of artificial intelligence, pattern recognition (Mitatha et al 2003), data mining (Munakata 1997), decision analysis (El-Feky and Abou-El-Enien 2019; Singh and Huang 2020; Wei 2003), medicine (Fibak et al 1986; Pawlak et al 1986), civil engineering (Arciszewski and Ziarko 1999), and transportation (Akilbasha et al 2017; Das et al 2016; Osman et al 2016; Roy et al 2018). Rough concept is generally used solving the multi-objective programming (Abdelwahed Khalifa 2018; Garai et al 2019; Hamzehee et al 2016; Saad et al 2014), linear fractional programming (Omran et al 2016) problems, and game theory (Ammar and Brikaa 2019; Brikaa et al 2021). Youness (2006) and Osman et al (2011) proposed mathematical programming problems in rough environment, called rough programming problems, and defined two different solutions which are surely optimal and possibly optimal. Hamzehee et al (2014) proposed an approach for solving an LP problem in which all or some coefficients were rough intervals. In the solution process, two distinct LP problems were constructed for the lower and upper approximation of rough intervals, and two new solutions, i.e., completely and rather satisfactory solutions, were obtained. Emam et al (2016) proposed an interactive model based on interval and slice-sum methods for the solution of large-scale three-level fully rough integer LP problems.

Since there are many different types to represent uncertainty such as randomness, fuzziness, and roughness, researchers present novel techniques for hybrid uncertain scenarios where fuzziness and roughness exist. Thus, fuzziness and roughness play an important role among types of uncertainty programming problems to avoid losing uncertain information. Dubois and Prade (1990) presented the fuzzification of rough sets, then the number of studies combining fuzziness and roughness increased, and the fuzzy rough sets were defined. Liu and Liu (2009) presented some definitions of fuzzy rough numbers. Ammar and Muamer (2016) introduced a solution algorithm for the fuzzy rough linear fractional programming problems with fuzzy rough numbers and used the decomposition algorithm to obtain a fuzzy rough optimal solution. Ammar and Eljerbi (2019) applied the fuzzy rough approach to the multi-objective integer linear fractional programming problems in which all the parameters and variables were fuzzy rough numbers. Saad and Fathy (2019) solved an LP problem with rough interval coefficients in fuzzy environment. In this type of problem, the variables and right-hand side parameters were fuzzy numbers, whereas the coefficients were rough intervals. Ammar and Emsimir (2021) suggested an algorithm to solve triangular fuzzy rough integer programming problems and found rough-valued optimal solutions and rough integer decision variables. In the problem, all parameters and decision variables were triangular fuzzy rough numbers. Moreover, the notion of fuzzy roughness was considered in several decision-making problems. For example, Brikaa et al (2019) presented an algorithm for solving constrained matrix games having fuzzy rough numbers. Pamučar et al (2019) presented an approach for the treatment of imprecision and uncertainty based on interval-valued fuzzy rough numbers and developed a multi-criteria decision-making model based upon the proposed approach. Zhu et al (2020) proposed a group decision-making structure for design concept evaluation by integrating AHP and TOPSIS having fuzzy rough numbers.

The aim of this paper is developing an algorithm for the solution of FRLP problems having fuzzy rough numbers to tackle the uncertainty in decision-making problems. In the problem, the parameters are triangular fuzzy rough numbers, and the variables are in the triangular fuzzy form. To solve the FRLP problem, two FFLP problems are obtained from the FRLP problem having the lower and the upper approximation approximations. Therefore, two different fuzzy optimal solutions are found by solving each FFLP problem, and fuzzy objective function values are evaluated by using these solutions. Thus, minimum and maximum supports of the fuzzy objective function values are determined. The proposed algorithm generates the worst and the best compromise solutions for DM to specify the decision bounds through these supports. Since the algorithm proposed in the study (Ahlatcioglu Ozkok et al 2016) deals with the infeasibility case of FFLP problems, it is also an advantage for finding a solution to the FRLP problem. If the FFLP problems obtained from the FRLP problem have fuzzy optimal solutions, the solution to the FRLP problem can be found. Else, the solution to the FRLP problem can be obtained by finding the approximate fuzzy optimal solution for each FFLP problem. Moreover, different fuzzy objective function values are determined from the solutions of lower and upper approximations. These fuzzy values are compared considering their supports, and fuzzy compromise solutions, which are called the best and the worst, are generated according to the direction of the optimization.

The paper is organized as follows: In Sect. 2, basic definitions of fuzzy numbers, fuzzy rough numbers, FRLP and FFLP problems are presented. Section 3 introduces the proposed algorithm in steps while some numerical experiments are solved in Sect. 4. Finally, the conclusion and future directions are stated in Sect. 5.

2 Preliminaries

Definition 1 Let $A \subset \mathbb{R}$ be a qualitative value, and $A^L = [a^{LL}, a^{LU}]$ and $A^U = [a^{UL}, a^{UU}]$ are lower and upper approximations intervals of A, respectively. $A = (A^L, A^U) = ([a^{LL}, a^{LU}], [a^{UL}, a^{UU}])$ is called a rough interval if the following properties are satisfied:

- If $x \in A^L$ then $x \in A$ (i.e., x is surely in A).
- If $x \notin A^U$ then $x \notin A$ (i.e., x is not surely in A).
- If $x \in A^U$ then x is possibly in A.

It is important to note that the lower approximation interval is defined inside of the upper approximation interval, i.e., $A^L \subseteq A^U$, and they are not complement of each other.

The illustration of a rough interval is presented in Fig. 1.

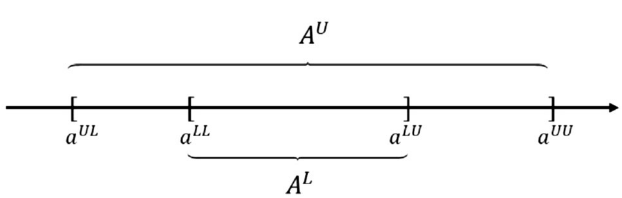

Fig. 1 A rough interval

Definition 2 Let $\tilde{A}$ be a convex normalized fuzzy set with the piecewise continuous membership function $\mu_{\tilde{A}} : \mathbb{R} \to [0, 1]$. The ordered triplet $\tilde{A} = (a^L, a^M, a^U)$ is a triangular fuzzy number (TFN) if its membership function is

$$\mu_{\tilde{A}}(r) = \begin{cases} \dfrac{r - a^L}{a^M - a^L}, & a^L \le r < a^M \\ \dfrac{r - a^U}{a^M - a^U}, & a^M \le r \le a^U \\ 0, & \text{otherwise.} \end{cases}$$

If $a^L \ge 0$, then $\tilde{A}$ is a non-negative TFN. The support of $\tilde{A}$ is the distance between the left and right components of the TFN, i.e., $supp(\tilde{A}) = |a^U - a^L|$.

The illustration of a TFN is presented in Fig. 2.

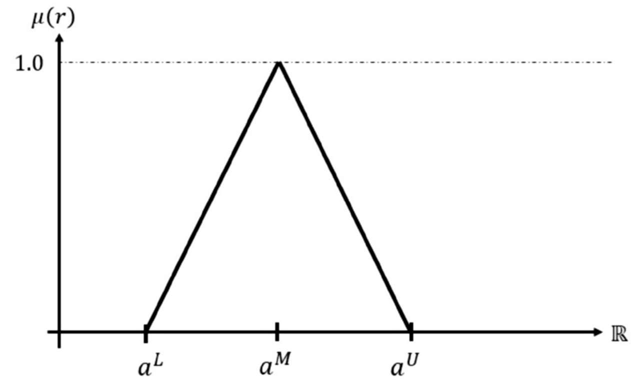

Fig. 2 A triangular fuzzy number

Definition 3 Let $\tilde{A}^R$ be a normalized convex fuzzy rough set of $\mathbb{R}$ with the piecewise continuous membership function. $\tilde{A}^R = (\tilde{A}^L, \tilde{A}^U)$ is a triangular fuzzy rough number (TFRN) in $\mathbb{R}$, where $\tilde{A}^L = (a^{LL}, a^{LM}, a^{LU})$ and $\tilde{A}^U = (a^{UL}, a^{UM}, a^{UU})$ are TFNs whose membership functions are defined as $\mu_{\tilde{A}^L} : \mathbb{R} \to [0, 1]$ and $\mu_{\tilde{A}^U} : \mathbb{R} \to [0, 1]$, respectively, and $\mu_{\tilde{A}^L}(r) \le \mu_{\tilde{A}^U}(r) \quad \forall r \in \mathbb{R}$. Here, $a^{LL}, a^{LM}, a^{LU}, a^{UL}, a^{UM}$ and a^{UU} are real numbers, and it should be noticed that $a^{UL} \le a^{LL} \le a^{LM} = a^{UM} \le a^{LU} \le a^{UU}$. The membership function of the TFRN is defined as follows:

$$\mu_{\tilde{A}^R}(r) = \begin{cases} \mu_{\tilde{A}^L}(r) = \begin{cases} \dfrac{r - a^{LL}}{a^{LM} - a^{LL}}, & a^{LL} \le r \le a^{LM} \\ \dfrac{r - a^{LU}}{a^{LM} - a^{LU}}, & a^{LM} \le r \le a^{LU} \\ 0, & \text{otherwise} \end{cases} \\ \mu_{\tilde{A}^U}(r) = \begin{cases} \dfrac{r - a^{UL}}{a^{UM} - a^{UL}}, & a^{UL} \le r \le a^{UM} \\ \dfrac{r - a^{UU}}{a^{UM} - a^{UU}}, & a^{UM} \le r \le a^{UU} \\ 0, & \text{otherwise.} \end{cases} \end{cases}$$

From the property of fuzzy rough numbers, $\tilde{A}^L$ and $\tilde{A}^U$ are lower and upper approximation TFNs of $\tilde{A}^R$, and $\tilde{A}^L \subset \tilde{A}^U$. The illustration of a TFRN is presented in ?tic=?>Fig. 3.

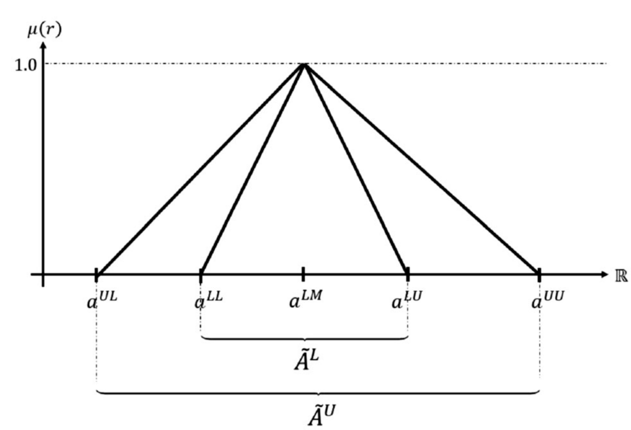

Fig. 3 A triangular fuzzy rough number

Let $\tilde{A}^R$ and $\tilde{B}^R$ be two nonnegative TFRNs given as $\tilde{A}^R = (\tilde{A}^L, \tilde{A}^U) = ((a^{LL}, a^{LM}, a^{LU}), (a^{UL}, a^{UM}, a^{UU}))$ and $\tilde{B}^R = (\tilde{B}^L, \tilde{B}^U) = ((b^{LL}, b^{LM}, b^{LU}), (b^{UL}, b^{UM}, b^{UU})]$. Arithmetic operations on these TFRNs are defined as follows:

Summation:

$$\begin{aligned}\tilde{A}^R + \tilde{B}^R &= (\tilde{A}^L + \tilde{B}^L, \tilde{A}^U + \tilde{B}^U)\\ &= \big((a^{LL} + b^{LL}, a^{LM} + b^{LM}, a^{LU} + b^{LU}),\\ &= (a^{UL} + b^{UL}, a^{UM} + b^{UM}, a^{UU} + b^{UU})\big).\end{aligned}$$

Subtraction:

$$\begin{aligned}\tilde{A}^R - \tilde{B}^R &= (\tilde{A}^L - \tilde{B}^L, \tilde{A}^U - \tilde{B}^U)\\ &= \big((a^{LL} - b^{LU}, a^{LM} - b^{LM}, a^{LU} - b^{LL}),\\ &= (a^{UL} - b^{UU}, a^{UM} - b^{UM}, a^{UU} - b^{UL})\big).\end{aligned}$$

Multiplication:

$$\begin{aligned}\tilde{A}^R \tilde{B}^R &= (\tilde{A}^L \tilde{B}^L, \tilde{A}^U \tilde{B}^U)\\ &= \big((a^{LL} b^{LL}, a^{LM} b^{LM}, a^{LU} b^{LU}),\\ &= (a^{UL} b^{UL}, a^{UM} b^{UM}, a^{UU} b^{UU})\big).\end{aligned}$$

Division:

$$\begin{aligned}\tilde{A}^R / \tilde{B}^R &= (\tilde{A}^L / \tilde{B}^L, \tilde{A}^U / \tilde{B}^U)\\ &= \big((a^{LL}/b^{LU}, a^{LM}/b^{LM}, a^{LU}/b^{LL}),\\ &= (a^{UL}/b^{UU}, a^{UM}/b^{UM}, a^{UU}/b^{UL})\big),\end{aligned}$$

where $0 \notin \tilde{B}^R$.

Definition 4 Let $\tilde{A}^R = (\tilde{A}^L, \tilde{A}^U)$ and $\tilde{B}^R = (\tilde{B}^L, \tilde{B}^U)$ be two TFRNs. Order relation between these fuzzy rough numbers are defined as follows:

- $\tilde{A}^R \leq \tilde{B}^R$ iff $\tilde{A}^L \leq \tilde{B}^L$ and $\tilde{A}^U \leq \tilde{B}^U$
- $\tilde{A}^R \geq \tilde{B}^R$ iff $\tilde{A}^L \geq \tilde{B}^L$ and $\tilde{A}^U \geq \tilde{B}^U$
- $\tilde{A}^R = \tilde{B}^R$ iff $\tilde{A}^L = \tilde{B}^L$ and $\tilde{A}^U = \tilde{B}^U$.

Consider a FRLP problem having n fuzzy variables and m mixed constraints. The FRLP problem with TFRNs can be defined

$$Min\ (or\ Max)\ \sum_{j=1}^{n} (\tilde{c}_j^L, \tilde{c}_j^U)\tilde{x}_j \tag{1a}$$

s.t.

$$\sum_{j=1}^{n} (\tilde{a}_{ij}^L, \tilde{a}_{ij}^U)\tilde{x}_j \{\geq, =, \leq\} (\tilde{b}_i^L, \tilde{b}_i^U), \forall i \in \{1, 2, .., m\}, \tag{1b}$$

where $(\tilde{c}_j^L, \tilde{c}_j^U), (\tilde{a}_{ij}^L, \tilde{a}_{ij}^U)$, and $(\tilde{b}_i^L, \tilde{b}_i^U)$ are TFRNs, and $\tilde{x}_j$ $\forall j \in \{1, 2, ..., n\}$, are nonnegative triangular fuzzy variables. The solution of the FRLP problem is in the fuzzy form whereas the objective function value is in the fuzzy rough form. Thus, the objective function value is denoted by $z(\tilde{x}) = (\tilde{z}^L, \tilde{z}^U)$, where $\tilde{z}^L$ and $\tilde{z}^U$ are lower and upper approximation TFNs, respectively.

Definition 5 (Osman et al 2011) Solutions of the FRLP problem (1) have different feasibility degrees as explained below:

- $\tilde{x}$ is surely fuzzy feasible solution if it is in the lower approximation of the feasible set,
- $\tilde{x}$ is possibly feasible solution if it is in the upper approximation of the feasible set,
- $\tilde{x}$ is surely not feasible solution if it is not in the upper approximation of the feasible set.

A FFLP problem having n fuzzy variables and m mixed constraints is presented as

$$Min\ (or\ Max)\ \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \tag{2a}$$

s.t.

$$\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \{\geq, =, \leq\} \tilde{b}_i, \quad \forall i \in \{1, 2, ..., m\}, \tag{2b}$$

where $\tilde{c}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$ are TFNs, and $\tilde{x}_j$ $\forall j \in \{1, 2, ..., n\}$ are nonnegative triangular fuzzy variables. Both the solution and the objective function value of the FFLP problem will be in fuzzy form.

Definition 6 Fuzzy feasible solution of the FFLP problem (2) is $\tilde{X} = [\tilde{x}_j]_{n \times 1}$ if it satisfies the following features:

- Each $\tilde{x}_j$ is a nonnegative TFN,
- $\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \{\geq, =, \leq\} \tilde{b}_i$, $\forall i \in \{1, 2, ..., m\}$.

Definition 7 Let $\tilde{A} = (a^L, a^M, a^U)$ be a TFN. Yager's ranking function can be applied to $\tilde{A}$ such as

$$\mathbf{R}(\tilde{A}) = \frac{a^L + 2a^M + a^U}{4},$$

where $\mathbf{R}(\tilde{A})$ is a real value. Therefore, any ranking function $\mathbf{R}$ is mapped from the set of fuzzy numbers to the set of real numbers and has a natural order.

Definition 8 (Ahlatcioglu Ozkok et al 2016) Fuzzy optimal solution of the FFLP problem (2) is $\tilde{X}^* = [\tilde{x}_j^*]_{n \times 1}$ if it satisfies the following features:

- $\tilde{X}^*$ is a fuzzy feasible solution,

- If there exist any nonnegative $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ satisfying all the constraints, then $\mathbf{R}(\tilde{z}^*) \leq \mathbf{R}(\tilde{z})$ in the case of minimization and $\mathbf{R}(\tilde{z}^*) \geq \mathbf{R}(\tilde{z})$ in the case of maximization.

Definition 9 (Ahlatcioglu Ozkok et al 2016) Approximate fuzzy optimal solution of the FFLP problem (2) is $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ if it satisfies the following features:

- $\tilde{X}$ is not a fuzzy feasible solution,
- for at least one equality constraint:
  - the left component of TFN in the right-hand side of the constraint is less than or equal to the right component of TFN in the left-hand side of the constraint;
  - the right component of the TFN in the right-hand side of the constraint is greater than or equal to the left component of the TFN in the left-hand side of the constraint.

Definition 10 A compromise solution is a feasible solution found by mutual concessions and optimizes the DM's utility function considering all criteria.

3 Proposed algorithm

This section presents the proposed algorithm for solving a FRLP problem. The steps of the algorithm are given as follows:

Step 1. Load the FRLP problem (1).

Step 2. Separate the FRLP problem into the following FFLP problems: the lower approximation of the FRLP problem is

$$Min\ (or\ Max)\ \tilde{z}^L = \sum_{j=1}^{n} \tilde{c}_j^L \tilde{x}_j \tag{3a}$$

s.t.

$$\sum_{j=1}^{n} \tilde{a}_{ij}^L \tilde{x}_j \{\geq, =, \leq\} \tilde{b}_i^L, \quad \forall i \in \{1, 2, ..., m\}, \tag{3b}$$

and the upper approximation of the FRLP problem is

$$Min\ (or\ Max)\ \tilde{z}^U = \sum_{j=1}^{n} \tilde{c}_j^U \tilde{x}_j \tag{4a}$$

s.t.

$$\sum_{j=1}^{n} \tilde{a}_{ij}^U \tilde{x}_j \{\geq, =, \leq\} \tilde{b}_i^U, \quad \forall i \in \{1, 2, ..., m\}. \tag{4b}$$

Step 3. Solve problems (3) and (4) by any FFLP solution method from the literature. It should be noted that the solution algorithm proposed in the study (Ahlatcioglu Ozkok et al 2016) is utilized here. The algorithm follows the steps given below:

- Convert the inequality constraints of the FFLP problem into equality forms by adding arbitrary TFNs introduced for the left- and right-hand sides, respectively.
- Construct a crisp LP problem by using the fuzzy equality concept in the constraints and a ranking function in the objective function.
- Solve the crisp LP problem. If there is an optimal solution for the LP problem, a fuzzy optimal solution is found for the FFLP problem. If the solution is infeasible, introduce new arbitrary TFNs for only the equality constraints of the FFLP problem and add them to the left and right sides of these constraints. Then, solve the rearranged crisp LP again and find the approximate fuzzy optimal solution.

The (approximate) fuzzy optimal solution of the FFLP problem (3) is $\tilde{x}^{L*}$, and the (approximate) fuzzy optimal solution of the FFLP problem (4) is $\tilde{x}^{U*}$.The (approximate) fuzzy optimal solutions of the FFLP problems (3) and (4) are named as the surely (approximate) feasible and possibly (approximate) feasible solutions to the FRLP problem (1), respectively.

Step 4. Determine the (approximate) fuzzy optimal values of the FFLP problems (3) and (4), i.e. $\tilde{z}^{L*} = \tilde{z}^L(\tilde{x}^{L*})$ and $\tilde{z}^{U*} = \tilde{z}^U(\tilde{x}^{U*})$, respectively. Then, evaluate $\tilde{z}_*^L = \tilde{z}^L(\tilde{x}^{U*})$ and $\tilde{z}_*^U = \tilde{z}^U(\tilde{x}^{L*})$ by substituting the possibly and surely (approximate) feasible solutions into the objective function of the FFLP problem (3) and (4), respectively.

Since two different (approximate) fuzzy optimal solutions are obtained, two different solutions to the FRLP problem will be found.

Step 5. Calculate the supports of each fuzzy objective function value by $min_{supp} = \{\tilde{z}^{L*}, \tilde{z}^{U*}, \tilde{z}_*^U, \tilde{z}_*^L\}$ and $max_{supp} = \{\tilde{z}^{L*}, \tilde{z}^{U*}, \tilde{z}_*^U, \tilde{z}_*^L\}$ and check the followings:

- The fuzzy objective function value having the minimum support corresponds to the lower approximation of a fuzzy rough compromise value, and this value can be called a *lower-min-TFN*. The fuzzy objective function value having the same middle component with *lower-min-TFN* is assigned to the upper approximation of this fuzzy rough compromise value, and this value can be called an *upper-min-TFN*. Therefore, one of the fuzzy rough compromise solutions can be found as *(lower-min-TFN, upper-min-TFN)*.
- The fuzzy objective function value having the maximum support corresponds to the upper approximation of another fuzzy rough compromise value, and this value can be called a *upper-max-TFN*. The fuzzy objective function value having the same middle

component with *upper-max-TFN* is assigned to the lower approximation of this fuzzy rough compromise value, and this value can be called an *lower-max-TFN*. Therefore, other fuzzy rough compromise solution can be found as *(lower-max-TFN, upper-max-TFN)*.

Step 6. Specify the best and the worst fuzzy compromise solutions according the direction of the optimization:

- In the case of maximization, the fuzzy optimal solution of the FFLP problem having the maximum support among the fuzzy objective function values is the best fuzzy compromise solution of the FRLP problem (1), and the other fuzzy optimal solution is the worst fuzzy compromise solution. In other words, *(lower-max-TFN, upper-max-TFN)* will be the best, and *(lower-min-TFN, upper-min-TFN)* will be the worst fuzzy compromise solution to the FRLP problem, respectively.
- In the case of minimization, the fuzzy optimal solution of the FFLP problem having the minimum support among the fuzzy objective function values is the best fuzzy compromise solution of the FRLP problem (1), and the other fuzzy optimal solution is the worst fuzzy compromise solution. Hence, *(lower-min-TFN, upper-min-TFN)* will be the best, and *(lower-max-TFN, upper-max-TFN)* will be the worst fuzzy compromise solution to the FRLP problem, respectively.

The flowchart of the algorithm is presented in Fig. 4.

It should be noted that the proposed algorithm has the advantage of producing two compromise solutions by separating the FRLP problem into two FFLP problems. Here, the fuzzy compromise solutions are determined from the solution of the FFLP problems and called the best and the worst, respectively, according to the direction of the optimization. Thus, it presents more than one solution for the DM to adapt the solution in the best and worst cases. On the other hand, the algorithm applied for the solutions to FFLP problems uses the fuzzy equality definition in converting the inequalities into equalities and a ranking function in the objective function. It is important to express that the proposed algorithm finds an approximate fuzzy optimal solution for a FFLP problem under the limitation of available resources in real-life problems. Therefore, as an advantage of the algorithm, whether there is an infeasible solution, an approximate fuzzy optimal solution to this case can be found for each FFLP problem. This helps the DM to determine an approximate fuzzy compromise solution in the infeasibility case. Lastly, since fuzzy rough sets are mainly based on interval arithmetic, the lower approximation will be a subset of the upper approximation, and this would yield to find compromise solutions in any case.

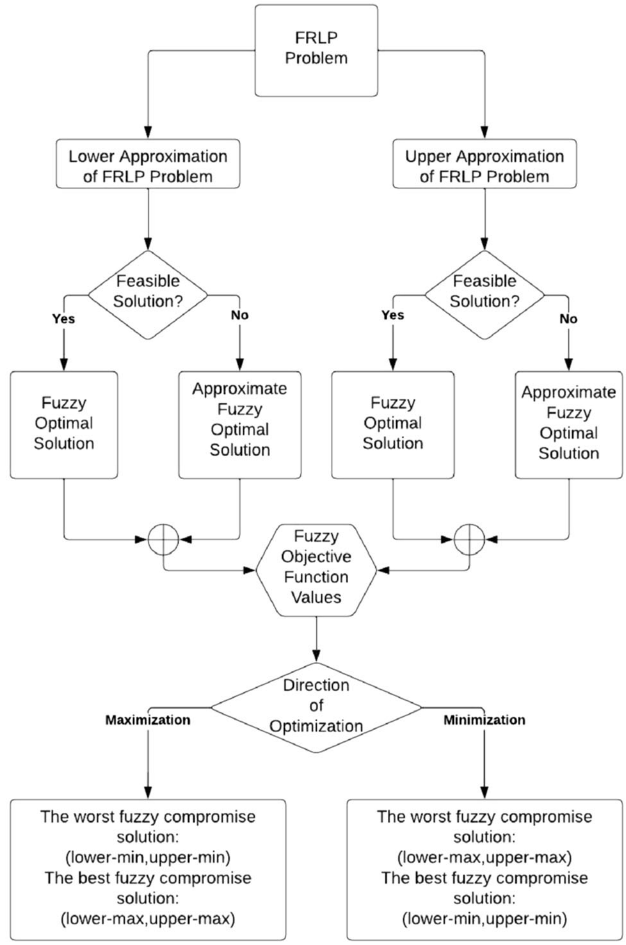

Fig. 4 Flowchart of the algorithm

4 Numerical examples

Example 1 Consider the following FRLP problem adapted from (Saad and Fathy 2019):

$$Max\ ((1,3,4),(0,3,6))\tilde{x}_1 + ((2,2,3),(1,2,4))\tilde{x}_2 \tag{5a}$$

s.t.

$$((2,3,4),(1,3,4))\tilde{x}_1 + ((3,4,5),(2,4,6))\tilde{x}_2 \\ \leq (6,9,11), \tag{5b}$$

$$((3,3,4),(2,3,6))\tilde{x}_1 + ((1,2,3),(0,2,5))\tilde{x}_2 \\ \leq (4,8,10), \tag{5c}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are nonnegative triangular fuzzy variables.

To solve the problem (1), the FRLP problem is separated into two FFLP problems. Hence, the lower approximation of the FRLP problem is

$$Max\ \tilde{z}^L = (1,3,4)\tilde{x}_1 + (2,2,3)\tilde{x}_2 \tag{6a}$$

s.t.

$$(2,3,4)\tilde{x}_1 + (3,4,5)\tilde{x}_2 \leq (6,9,11), \quad (6b)$$

$$(3,3,4)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \leq (4,8,10), \quad (6c)$$

whereas the upper approximation of the FRLP problem is

$$Max\ \tilde{z}^U = (0,3,6)\tilde{x}_1 + (1,2,4)\tilde{x}_2 \quad (7a)$$

s.t.

$$(1,3,4)\tilde{x}_1 + (2,4,6)\tilde{x}_2 \leq (6,9,11), \quad (7b)$$

$$(2,3,6)\tilde{x}_1 + (0,2,5)\tilde{x}_2 \leq (4,8,10). \quad (7c)$$

The FFLP problem (6) is rewritten as below:

$$Max\ \tilde{z}^L = (1,3,4)\tilde{x}_1 + (2,2,3)\tilde{x}_2 \quad (8a)$$

s.t.

$$(2,3,4)\tilde{x}_1 + (3,4,5)\tilde{x}_2 + \tilde{l}_1 = (6,9,11) + \tilde{r}_1, \quad (8b)$$

$$(3,3,4)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + \tilde{l}_2 = (4,8,10) + \tilde{r}_2, \quad (8c)$$

where $\tilde{x}_1 = (x_{1L}, x_{1M}, x_{1U})$ and $\tilde{x}_2 = (x_{2L}, x_{2M}, x_{2U})$ are nonnegative triangular fuzzy variables, $\tilde{l}_1 = (l_{1L}, l_{1M}, l_{1U})$, $\tilde{l}_2 = (l_{2L}, l_{2M}, l_{2U})$, and $\tilde{r}_1 = (r_{1L}, r_{1M}, r_{1U})$, $\tilde{r}_2 = (r_{2L}, r_{2M}, r_{2U})$ are arbitrary TFNs added to the left and right sides, respectively. After converting the inequality constraints into equality form, using the fuzzy equality concept in the constraints and a ranking function in the objective function, the following crisp LP problem is constructed:

$$Max\ 0.25x_{1L} + 1.5x_{1M} + x_{1U} + 0.5x_{2L} + x_{2M} + 0.75x_{2U} \quad (9a)$$

s.t.

$$2x_{1L} + 3x_{2L} + l_{1L} = 6 + r_{1L}, \quad (9b)$$

$$3x_{1M} + 4x_{2M} + l_{1M} = 9 + r_{1M}, \quad (9c)$$

$$4x_{1U} + 5x_{2U} + l_{1U} = 11 + r_{1U}, \quad (9d)$$

$$3x_{1L} + x_{2L} + l_{2L} = 4 + r_{2L}, \quad (9e)$$

$$3x_{1M} + 2x_{2M} + l_{2M} = 8 + r_{2M}, \quad (9f)$$

$$4x_{1U} + 3x_{2U} + l_{2U} = 10 + r_{2U}, \quad (9g)$$

$$x_{1M} - x_{1L} \geq 0; \quad x_{1U} - x_{1M} \geq 0, \quad (9h)$$

$$x_{2M} - x_{2L} \geq 0; \quad x_{2U} - x_{2M} \geq 0, \quad (9i)$$

$$l_{1M} - l_{1L} \geq 0; \quad l_{1U} - l_{1M} \geq 0, \quad (9j)$$

$$l_{2M} - l_{2L} \geq 0; \quad l_{2U} - l_{2M} \geq 0, \quad (9k)$$

$$r_{1M} - r_{1L} \geq 0; \quad r_{1U} - r_{1M} \geq 0, \quad (9l)$$

$$r_{2M} - r_{2L} \geq 0; \quad r_{2U} - r_{2M} \geq 0, \quad (9m)$$

$$l_{1L} + 2l_{1M} + l_{1U} \geq r_{1L} + 2r_{1M} + r_{1U}, \quad (9n)$$

$$l_{2L} + 2l_{2M} + l_{2U} \geq r_{2L} + 2r_{2M} + r_{2U}. \quad (9o)$$

Solving the crisp LP problem (9), fuzzy optimal solution of the problem (6) is found $\tilde{x}_1 = (0, 0.792, 5.062)$ and $\tilde{x}_2 = (0.625, 0.625, 0.625)$, and the fuzzy optimal value is $\tilde{z}^{L*} = (1.25, 3.626, 22.123)$. This fuzzy optimal solution is the surely feasible solution of the FRLP problem (1). The surely feasible solution is substituted into the fuzzy objective function of the FFLP problem (7), and $\tilde{z}^U_* = (0.625, 3.626, 32.872)$ is obtained. Similarly, the FFLP problem (7) is solved, and the possibly feasible solution of the FRLP problem (1) is found $\tilde{x}_1 = (0, 1.618, 1.618)$ and $\tilde{x}_2 = (1.176, 1.176, 1.176)$. The fuzzy optimal value of (7) is $\tilde{z}^{U*} = (1.25, 3.626, 22.123)$, and the fuzzy objective function value is $\tilde{z}^L_* = (2.352, 7.206, 10)$ obtained by substituting the possibly feasible solution into the fuzzy objective function of the FFLP problem (6).

The fuzzy objective function values are shown in Fig. 5. After computing each fuzzy objective function value, their supports are compared to determine the fuzzy compromise solutions. Since the FRLP is a maximization problem, the fuzzy objective function value having the minimum support will be the lower approximation of the worst fuzzy rough compromise value. Because $min_{supp}\{\tilde{z}^{L*}, \tilde{z}^{U*}, \tilde{z}^L_*, \tilde{z}^U_*\} = \{20.873, 7.648, 13.236, 32.247\} = 7.648$, the worst fuzzy rough compromise value is ((2.352, 7.206, 10), (1.176, 7.206, 14.412)), and the worst fuzzy compromise solution is $\tilde{x}_1 = (0, 1.618, 1.618)$ and $\tilde{x}_2 = (1.176, 1.176, 1.176)$. The fuzzy objective function value having the maximum support will be the upper approximation of the best fuzzy rough compromise value. Since $max_{supp}\{\tilde{z}^{L*}, \tilde{z}^{U*}, \tilde{z}^L_*, \tilde{z}^U_*\} = 32.247$, the best fuzzy rough compromise value is ((1.25, 3.636, 22.123), (0.625, 3.626, 32.872)), and the best fuzzy compromise solution is $\tilde{x}_1 = (0, 0.792, 5.062)$ and $\tilde{x}_2 = (0.625, 0.625, 0.625)$.

In (Saad and Fathy 2019), the example had rough interval coefficients and fuzzy variables, and the rough optimal range for the problem was found ([10.4, 37.4], [3.7, 96]). Here, if the fuzzy rough compromise values are rewritten in interval form, the best compromise range is ([1.25, 22.123], [0.625, 32.872]), and the worst compromise range is ([2.352, 10], [1.176, 14.412]). For the given example, the minimum of the supports corresponds to the surely range of the worst fuzzy rough compromise solutions, and this range has no common points with the surely solution of the study (Saad and Fathy 2019). On the other hand, the maximum of the supports corresponds to the possibly range of the best fuzzy rough compromise solutions, and this range has a wide common range with the surely solution of the study (Saad and Fathy

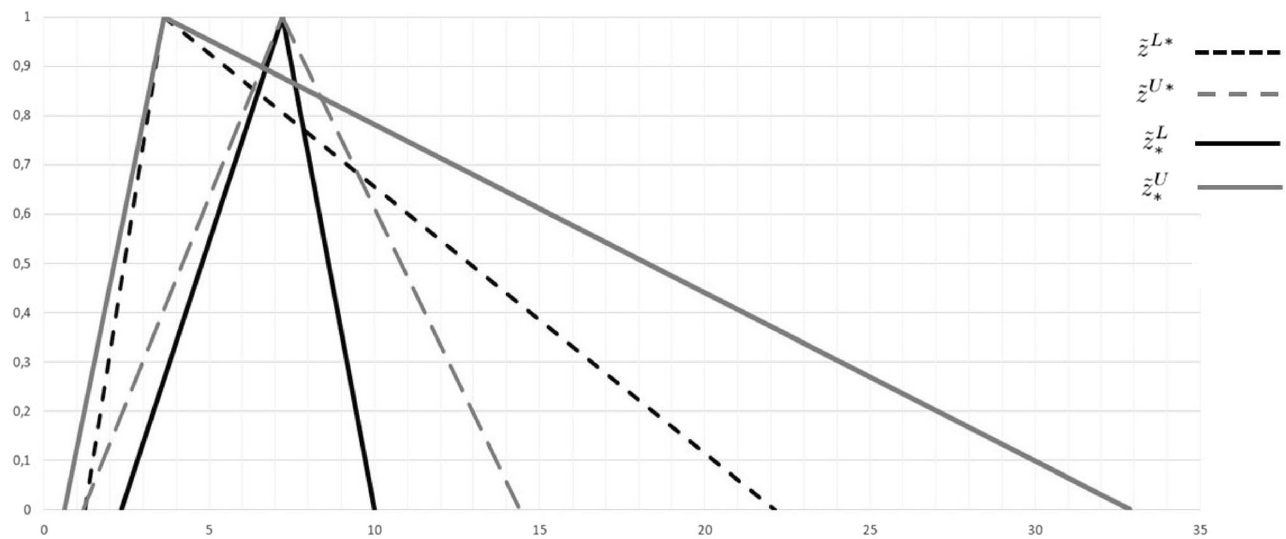

Fig. 5 Fuzzy objective function values in Example 1

2019). Therefore, it can be said that the intersection ranges of the intervals increase as they progress from worst to best.

Example 2 Consider the following FRLP problem adapted from the example in (Hamzehee et al 2014):

$$Max\ ((1,2,3),(0,2,5))\tilde{x}_1 + ((1,1,2),(1,1,3))\tilde{x}_2 \tag{10a}$$

s.t.

$$((1,2,3),(1,2,3))\tilde{x}_1 + ((2,3,4),(1,3,5))\tilde{x}_2 \leq ((7,8,9),(5,8,10)), \tag{10b}$$

$$((2,2,3),(1,2,5))\tilde{x}_1 + ((1,1,2),(0,1,6))\tilde{x}_2 \leq ((5,6,8),(3,6,9)), \tag{10c}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are nonnegative triangular fuzzy variables.

The FRLP problem (2) is separated into two FFLP problems. To find the surely feasible solution of the FRLP problem, the following FFLP problem is solved:

$$Max\ \tilde{z}^L = (1,2,3)\tilde{x}_1 + (1,1,2)\tilde{x}_2 \tag{11a}$$

s.t.

$$(1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 \leq (7,8,9), \tag{11b}$$

$$(2,2,3)\tilde{x}_1 + (1,1,2)\tilde{x}_2 \leq (5,6,8), \tag{11c}$$

and the fuzzy optimal solution is obtained as $\tilde{x}_1 = (0,3.571,3.571)$ and $\tilde{x}_2 = (0,0,0)$. The fuzzy optimal value of the FFLP problem (11) is $\tilde{z}^{L*} = (0,7.142,10.713)$. Similarly, to determine the possibly feasible solution of the FRLP problem (2), the following FFLP problem is solved:

$$Max\ \tilde{z}^U = (0,2,5)\tilde{x}_1 + (1,1,3)\tilde{x}_2 \tag{12a}$$

s.t.

$$(1,2,3)\tilde{x}_1 + (1,3,5)\tilde{x}_2 \leq (5,8,10), \tag{12b}$$

$$(1,2,5)\tilde{x}_1 + (0,1,6)\tilde{x}_2 \leq (3,6,9), \tag{12c}$$

and the fuzzy optimal solution is found $\tilde{x}_1 = (0,1.5,3.6)$ and $\tilde{x}_2 = (0,0,0)$, and the fuzzy optimal value is $\tilde{z}^{U*} = (0,3,18)$.

The surely feasible solution is substituted into the fuzzy objective function of the FFLP problem (12), and $\tilde{z}^U_* = (0,7.412,17.855)$ is determined. Also, the possibly feasible solution is substituted into the fuzzy objective function of the FFLP problem (11), and $\tilde{z}^L_* = (0,3,10.8)$ is evaluated. Since the objective function of the FRLP problem is maximization, the worst fuzzy compromise solution is $\tilde{x}_1 = (0,3.571,3.571)$ and $\tilde{x}_2 = (0,0,0)$, and the worst fuzzy rough compromise value is ((0, 7.142, 10.713), (0, 7.142, 17.855)). In the same way, the best fuzzy rough compromise solution is $\tilde{x}_1 = (0,1.5,3.6)$ and $\tilde{x}_2 = (0,0,0)$, and the best fuzzy rough compromise value is ((0, 3, 10.8), (0, 3, 18)).

In (Hamzehee et al 2014), the example had rough interval coefficients and crisp variables. Hence, the rough optimal range was found ([333.33, 480], [160, 2216.67]). By applying the proposed algorithm, the best and the worst fuzzy rough compromise values are rewritten in interval form, and then ([0, 10.8], [0, 18]) and ([0, 10.713], [0, 17.855]) are obtained, respectively.

Example 3 Consider the FRLP problem adapted from (Ahlatcioglu Ozkok et al 2016):

$$Max\ ((1,2,3),(0,2,4))\tilde{x}_1 + ((3,4,5),(2,4,6))\tilde{x}_2 \tag{13a}$$

s.t.

$$((-2,2,6),(-3,2,7))\tilde{x}_1 + ((1,2,3),(0,2,4))\tilde{x}_2 \leq ((-3,8,24),(-5,8,30)), \tag{13b}$$

$$((2,4,6),(1,4,8))\tilde{x}_1 + ((1,3,5),(0,3,6))\tilde{x}_2 \geq ((1,13,32),(0,13,40)), \tag{13c}$$

$$((-1,0,1),(-2,0,2))\tilde{x}_1 + ((-3,-2,-1),(-4,-2,2))\tilde{x}_2 = ((-14,-6,1),(-20,-6,5)), \tag{13d}$$

$$((-3,-1,2),(-4,-1,5))\tilde{x}_1 + ((1,2,4),(0,2,6))\tilde{x}_2 = ((-3,-2,-1),(-4,-2,2)), \tag{13e}$$

$$((-4,-3,-2),(-5,-3,1))\tilde{x}_1 + ((1,3,6),(0,3,8))\tilde{x}_2 = ((1,4,5),(0,4,8)), \tag{13f}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are nonnegative triangular fuzzy variables.

The FRLP problem (3) is separated into two FFLP problems. Since the solution of the lower approximation of the FRLP problem is infeasible, arbitrary fuzzy variables are added to both sides of the fuzzy equality constraints, and then the FFLP problem is rewritten as a crisp LP problem. After solving the LP problem, the approximate fuzzy optimal solution is found $\tilde{x}_1 = (0.875, 0.875, 0.875)$ and $\tilde{x}_2 = (3.75, 3.75, 3.75)$, that is the surely approximate solution of the FRLP problem (3), and the approximate fuzzy optimal value is $\tilde{z}^{L*} = (12.125, 16.75, 21.375)$. Likewise, the upper approximation of the FRLP problem (3) is solved, and the solution is found infeasible. Then, the approximate fuzzy optimal solution and approximate fuzzy optimal value are found $\tilde{x}_1 = (0.9, 0.9, 0.9)$ and $\tilde{x}_2 = (4.225, 4.225, 4.225)$, and $\tilde{z}^{U*} = (8.45, 18.7, 28.95)$, respectively.

The surely approximate solution is substituted into the fuzzy objective function of the upper approximation of the FRLP problem, and $\tilde{z}^U_* = (7.5, 16.75, 26)$ is obtained. Similarly, the possibly approximate solution is substituted into the fuzzy objective function of the lower approximation of the FRLP problem, and $\tilde{z}^L_* = (13.575, 18.7, 23.825)$ is evaluated. Since the objective function of the FRLP problem (13) is maximization, the best fuzzy rough compromise value is ((13.575, 18.7, 23.825), (8.45, 18.7, 28.95)), and the worst fuzzy rough compromise value is ((12.125, 16.75, 21.375), (7.5, 16.75, 26)).

5 Conclusion

In this paper, a novel solution algorithm for the FRLP problem having TFRN parameters and the TFN variables is presented. For the lower and upper approximations of the FRLP problem, two FFLP problems are constructed, and each is solved via an FFLP solution approach. The fuzzy optimal solutions are named as the surely feasible and possibly feasible solutions corresponding to the lower and upper approximations of the FRLP problem, respectively. Substituting the surely and possibly feasible solutions into the fuzzy objective function of each FFLP problem, fuzzy objective function values are determined, and these values are compared according to their supports. Then, the best and the worst fuzzy compromise solutions are specified considering the direction of the optimization. In the end, the best and the worst fuzzy rough compromise solutions are presented to the DM to make the final decision. An advantage of the proposed algorithm is that the DM has separate choices for the best and the worst situations. Also, it can be another advantage that finding approximate fuzzy solutions to the FFLP problems for infeasible cases and then presenting them to the DM as approximate fuzzy compromise solutions. Numerical experiments are illustrated to show the efficiency and effectiveness of the algorithm. Applying the algorithm to deal with multi-objective programming problems can be another subject for future research.

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Data availability statement All data generated or analyzed during this study are cited from the published articles (Hamzehee et al 2014; Ahlatcioglu Ozkok et al 2016; Saad and Fathy 2019).

Declarations

Conflict of interest The author declares that there is no conflict of interest.

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